음악의 기교에 대한 수학적 논의
Mathematical discussion of articulation in music

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ABSTRACT

Music is one great art with beauty and pattern. The study of music already has its underlying composition theory to have its own rules and tendency, which however still leaves questions that why it is the right way. Such analytical studies of modelling music began recently around decades to answer the questions by understanding or assessing musical elements to see the science in art. In this research, or small survey, I purposed to model harmony by use of mathematical geometry, but only in simplified manner, especially proposing and simulating some model for tension and harmony.

OBJECTIVES

- Propose mathematical model of harmony/progression; here only vertical harmony
- Numerical simulation with the model for simple example
- Observation and Analysis of superposed case; concrete example

Surface tension rule: $T_{diss}(y)$ = scale degree + inversion + non-harmonic tones (summed over all the pitch classes in y's span), where

scale degree = 1 if $\hat{3}$ or $\hat{5}$ in the melodic voice, 0 otherwise;

inversion = 2 if $\hat{3}$ or $\hat{5}$ in the bass, 0 otherwise;

non-harmonic tone = 3 if a pitch class is a diatonic non-chord tone, 4 if it is a chromatic non-chord tone, 0 otherwise.

Cf. Lerdahl's rating; surface tension

Model

Assumptions.

- Pythagorean argument; tones with simple integer ratio is considered harmonious
- Potential at each pair of tones
- Linear superposition
- Symmetry from equal temperament
- Articulations/melodies are given as perturbation of simple chords.
- Most music comes from tension and amount leap of the melody

$$|r-\frac{a}{b}| < \Delta = r\epsilon$$

$$|\frac{1}{r}-\frac{b}{a}|/\frac{1}{r_0} \approx \frac{\Delta}{r_0^2}\cdot r_0 = \frac{\Delta}{r_0} < \epsilon$$

T(1,1)=0

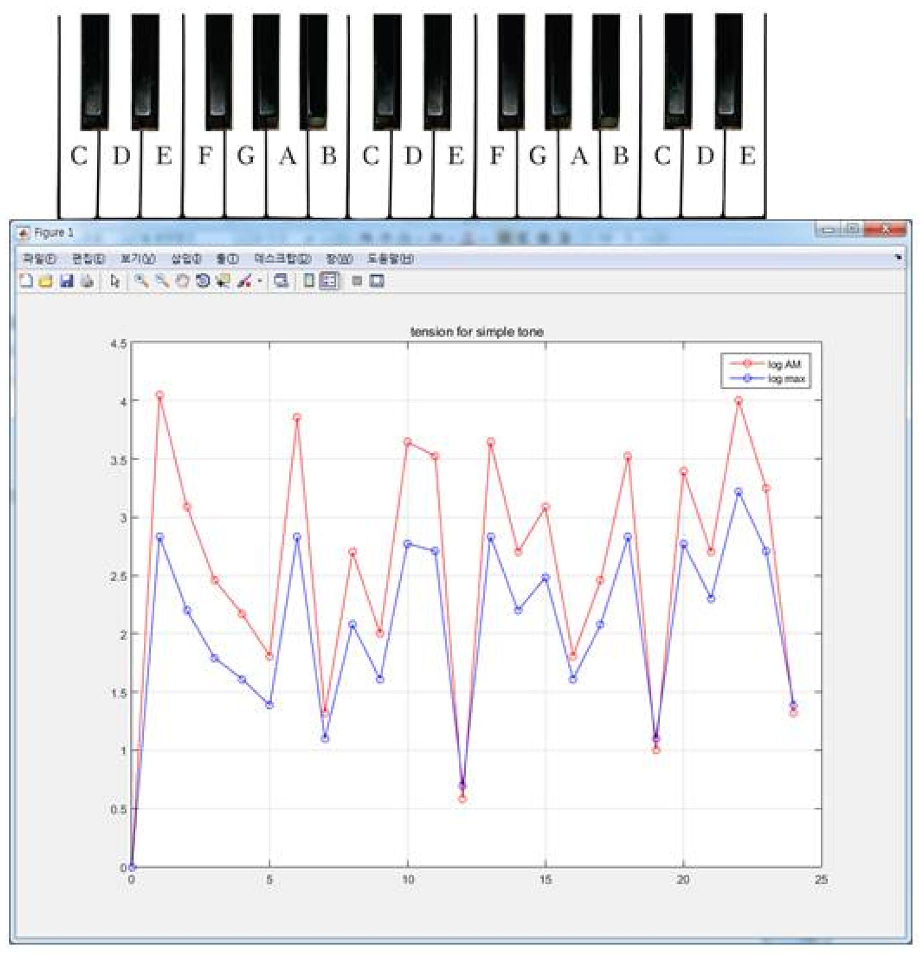
Linear $\quad T(m,n,l) = T(m,n) + T(n,l) + T(m,l)$

Symmetric

T(a,0)=0

$$T(a,b) = a+b$$
$$T(a,b) = \log\max(a,b) = \log a \vee b$$
$$T(a,b) = \log\frac{a+b}{2}$$

$$E(m,n) = \sum_{i=1,2,\dots}\sum_{j=1,2,\dots} A_iA_jE(im,jn) \approx \sum_{finite\ i,j's} A_iA_jE(im,jn)$$

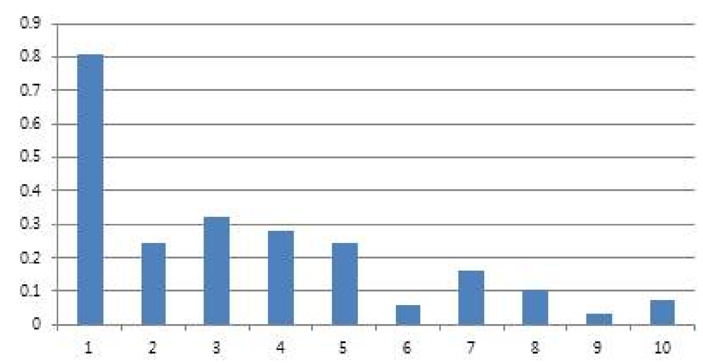


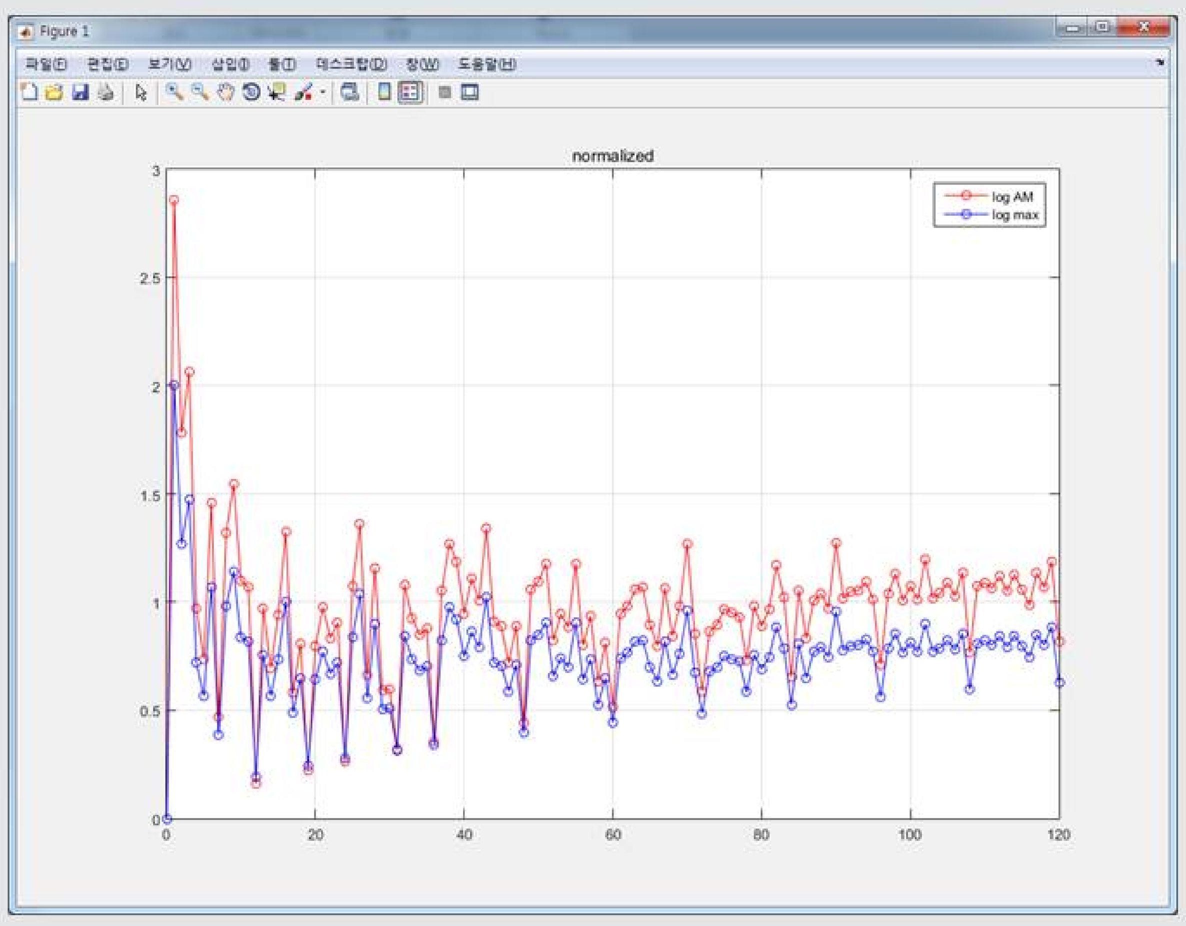
Application examples

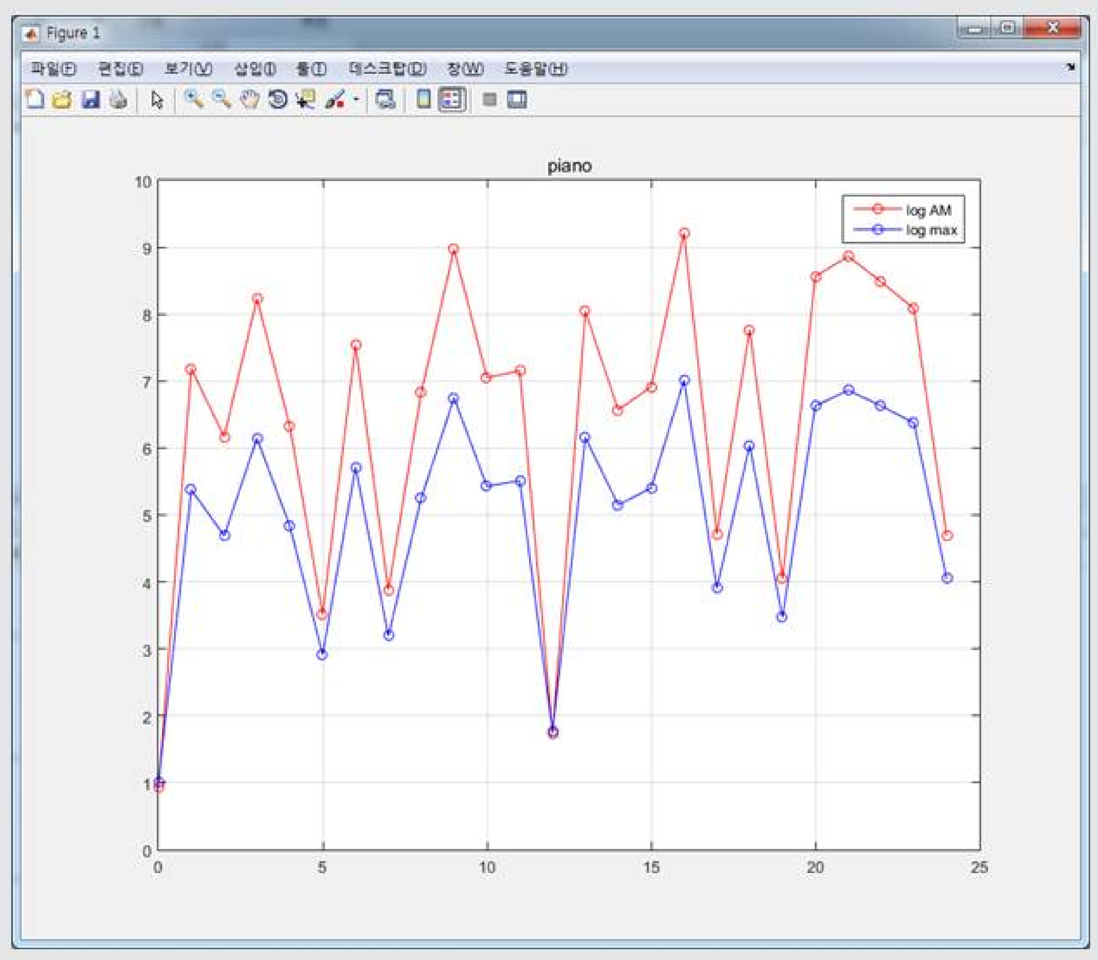
Remarks.

- Scales;
- Major, minor, pentatonic
- Diatonic / chromatic

Spectral Consideration (piano)







CONCLUSIONS

In this research, I proposed some intuitive and simple model with linearity assumption.
Though I intended to discuss dissonance through rationality and frequency as it sounds, still the result was frustrating and quite not connects to initial idea. Other simulations not included here with real music data also did not work in the right way, even ending up without any clues one can find.
This way fails to deal with numerical difficulty and perhaps it is right to follow traditional.

chords	tension	tuned
Maj	5.951285	17.31354
	6.967226	
	5.97728	
min	5.951285	
	5.97728	
	7.04029	
aug	7.04029	
dim	8.776844	23.51756
sus	6.216746	