Statistical Approach of Tetris Dynamics

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Introduction

TETRIS game has very intuitive and obvious rules. In this research, we generated the system which has same physics with the tetris game in virtual, and observed thermodynamic effects to understand the tetris world.

Tetris world has two interactions:

- block interaction : the interaction by blocks from outside.
- line interaction : breaking tiles when there are all occupied rows.

Question : all the TETRIS games have the width 10 world and length 4 blocks. Why so? Statistical approaches could solve that problem.

Statistical approach

- Hamiltonian

We introduced the Hamiltonian

$$H = \frac{1}{W}\sum_{i=1}^{W} s_i + \frac{1}{L}\text{stdev}(\{s_i\})$$

W represents the width of tetris world, s_i means the maximum height of i-site, and L means the length of the given blocks.

- Entropy

To calculate the thermodynamic potentials, such as Helmholtz free energy, we had to calculate the number of all possible states, and trace of them.

We introduced generating function whose n-th order coefficient corresponds to the number of possible states when n tiles are occupied.

$$\text{GF} = \left((1+x)^W - 1 - x^W\right)^H$$

The above equation is generating function when the tetris world is W by H. After that, we generated all possible $\{s_i\}$ set using mathematica when width, and volume are given and calculated entropy of that world.

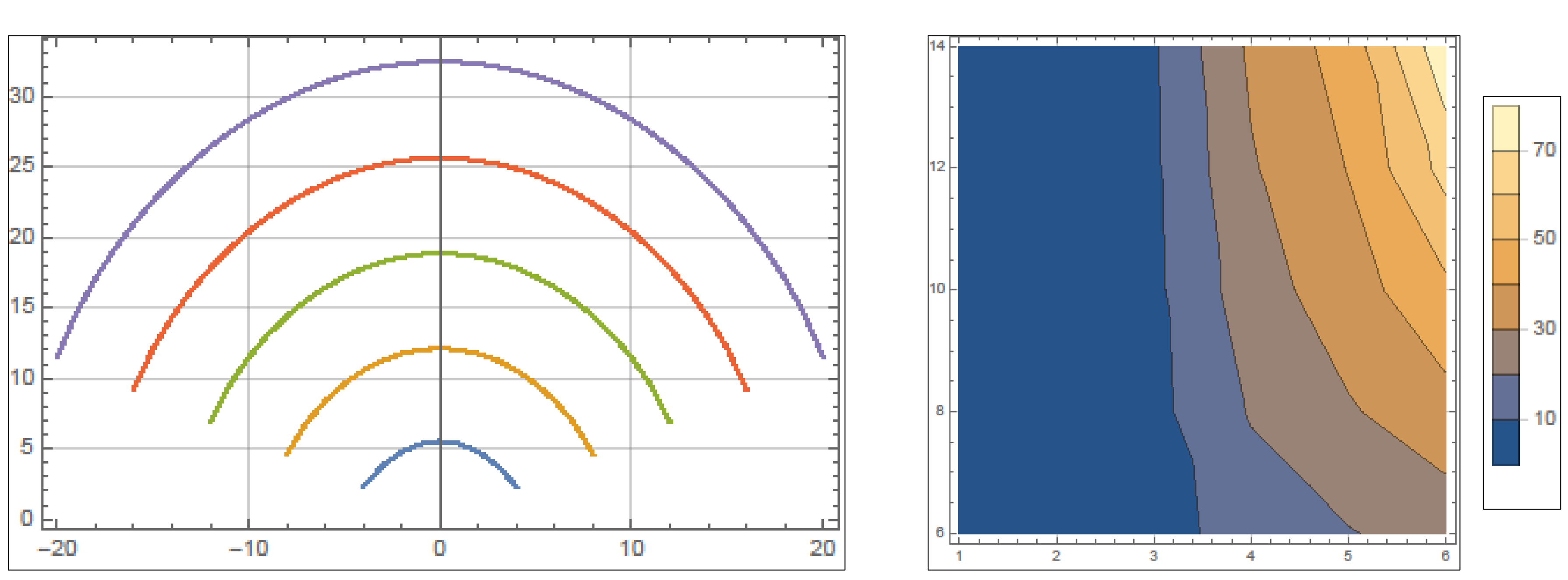

Fig.1 (a) Entropy vs (centered) weight (b) Entropy vs width and volume

- Free Energy

When free energy goes to zero, there would be phase transition. To see the critical temperature, we calculated Helmholtz free energy.

$$Z = \text{Tr}\left(e^{-\beta H}\right), \qquad F = -\frac{1}{\beta}\ln Z$$

We calculated the critical temperature which makes free energy be zero for various width, volume, and long.

Critical temperature vs width and volume

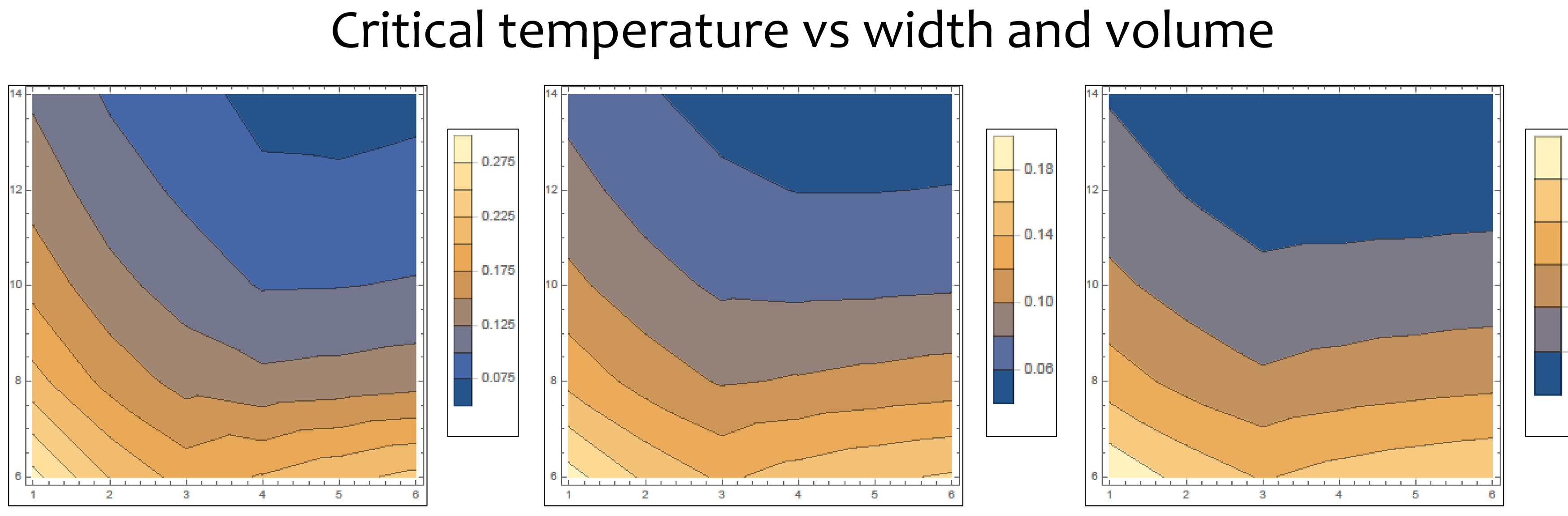

Fig.2 (a) $L = 1$ (b) $L = 2$ (c) $L = 3$

Simulation results

- Decision process of a block's drop position

A position of a generated block has a probability distribution of canonical ensemble, parametrized by a temperature of tetris system.

$$p \propto \exp(-\beta H)$$

In $T \to \infty$ limit, a block falls to a completely random position.

In $T \to 0$ limit, a block falls to a position that minimizes the Hamiltonian.

- Equilibrium confirmation

A system reached its equilibrium with $T = 0$ when $L < 4$.

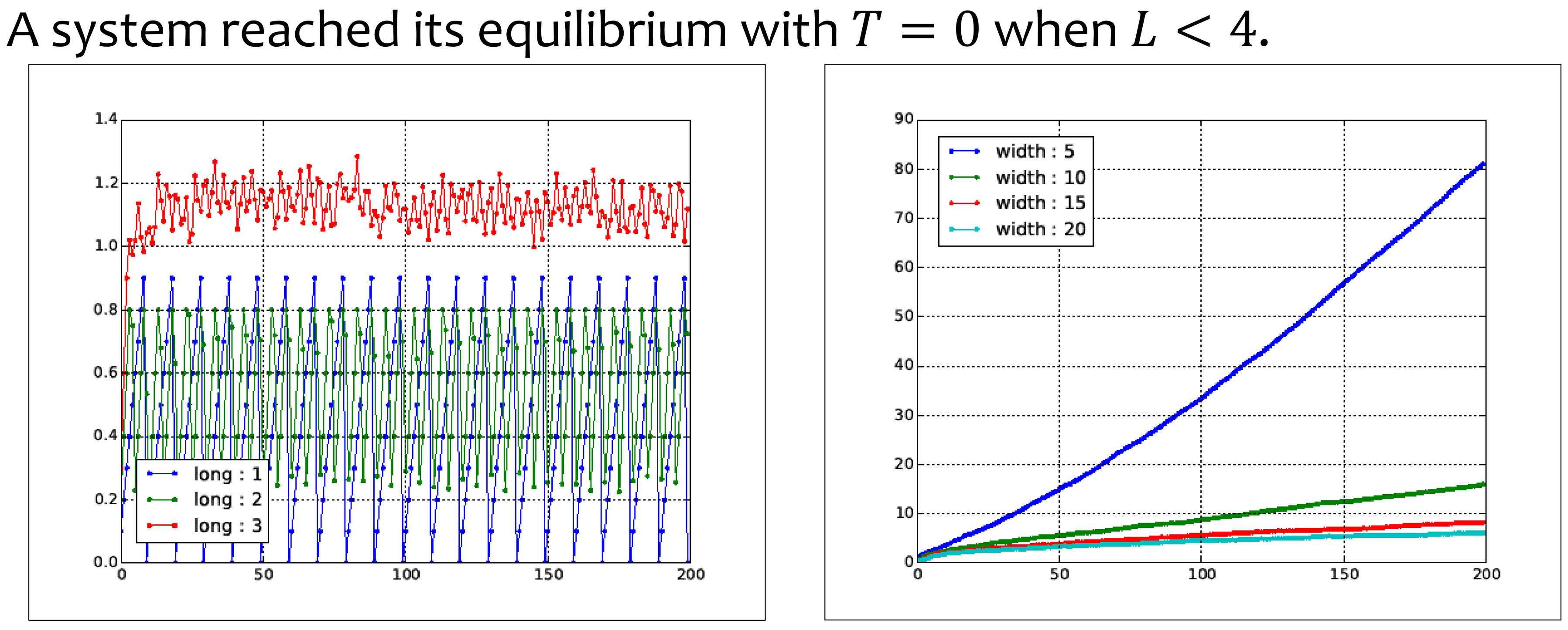

Fig.3 (a) Average height with $W = 10$ (b) Average height $L = 4$

- Critical temperature and critical exponent

When $T < T_c$, a system converges to its equilibrium. We seek to obtain the critical temperature T_c and the critical exponent α by fitting the equilibrium region with $c\,(T_c - T)^{-\alpha}$.

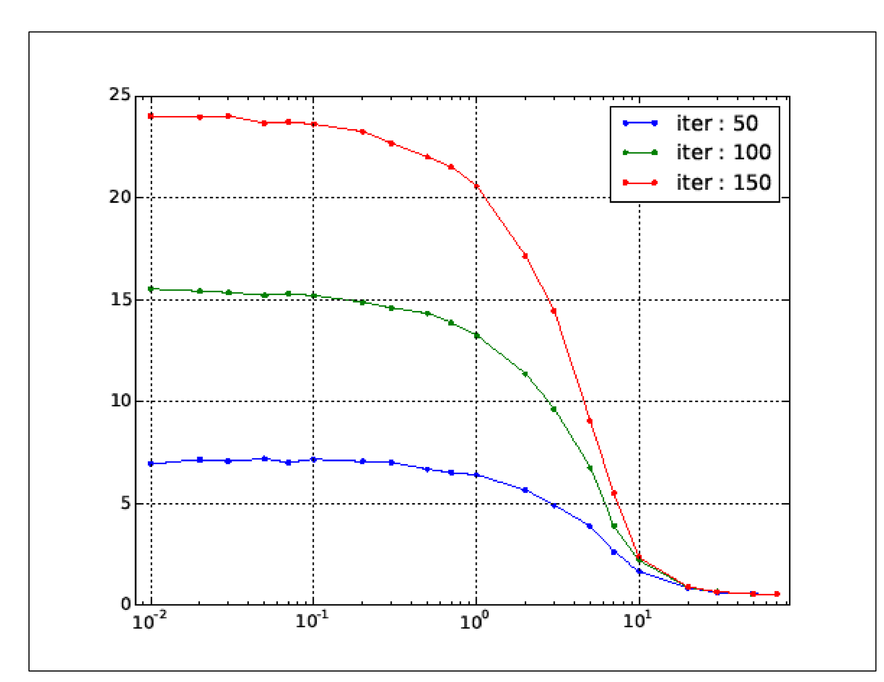

Fig.4 Average height against β

When $T > T_c$, the average height gradually increases with iteration. The critical temperature is seated around an order of $\mathcal{O}(10^{-2})$

Critical values vs width and volume

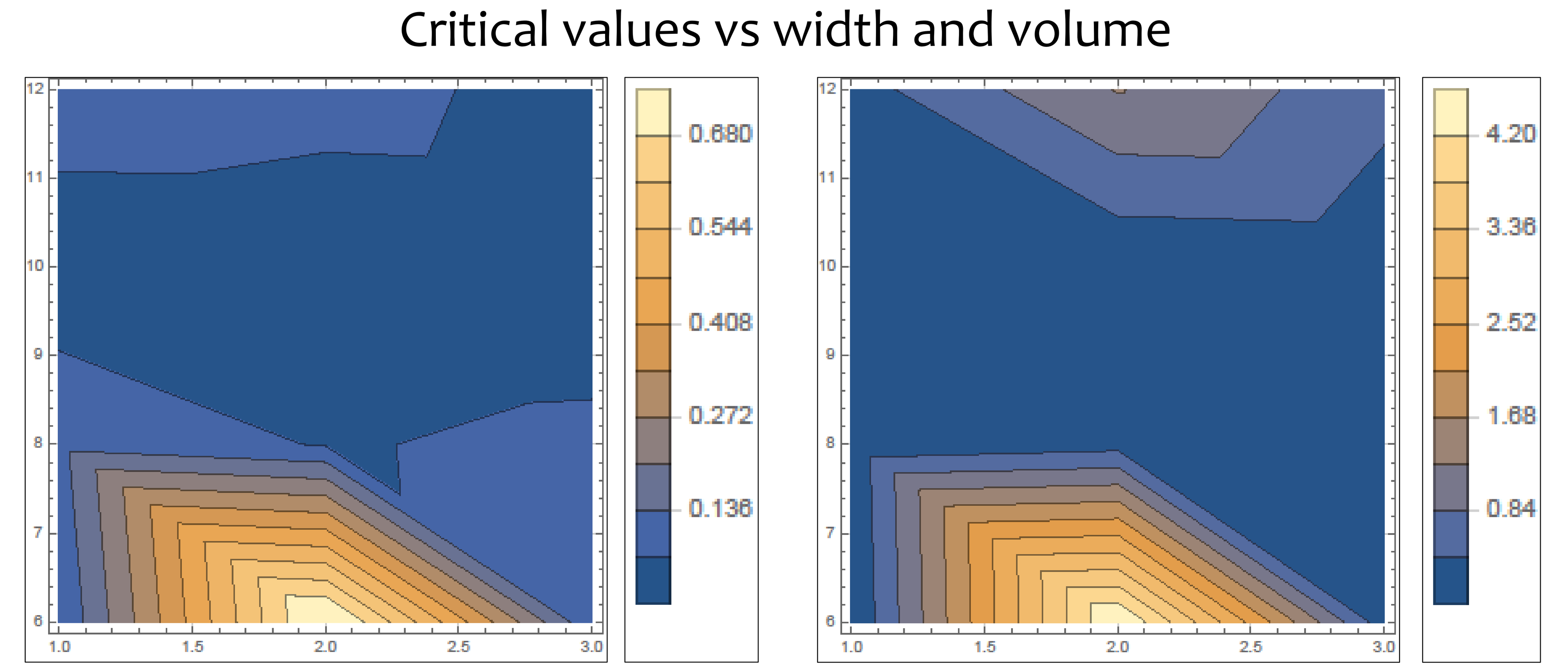

Fig.5 (a) Critical temperature (b) Critical exponent

Conclusion

Our Hamiltonian works well to let tetris system reach to the equilibrium when $L < 4$. We calculated the critical temperature and the critical exponent from the average height.

$L = 4$ makes the system be difficult to reach the equilibrium while it provides reasonable number of polyominoes.

$W = 10$ minimizes the critical temperature so that the system has much more fluctuations.

Reference

[1] J. M. Yeomans, *Statistical Mechanics of Phase Transitions*, 1992

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[3] Golomb, Solomon W., *Polyominoes*, 1994